

# Charged lepton decays from soft flavour violation in a two-Higgs doublet seesaw model

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$\int dk \prod$  Doktoratskolleg  
Particles and Interactions



# Overview

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## THE MODEL

- Neutrinos - right handed neutrinos
- Multi Higgs doublets
- Soft flavour violation
- Properties and advantages

2

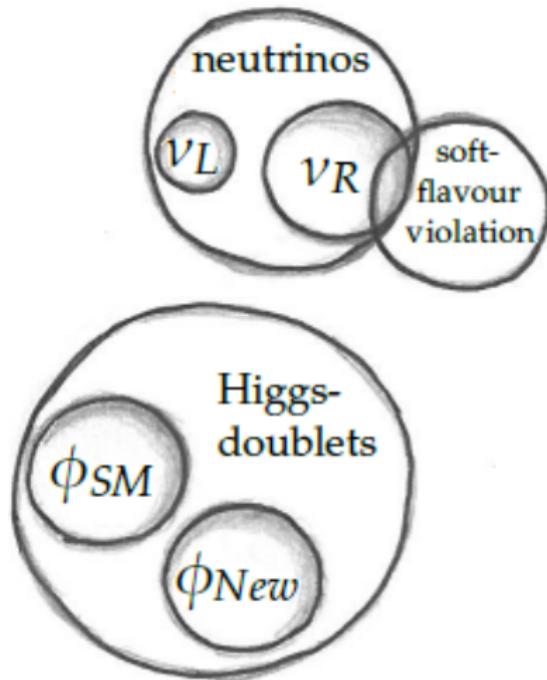
## CHARGED LEPTON DECAYS

- Suppression of radiative corrections
- Free Parameter
- Magnetic dipole moments
- Results

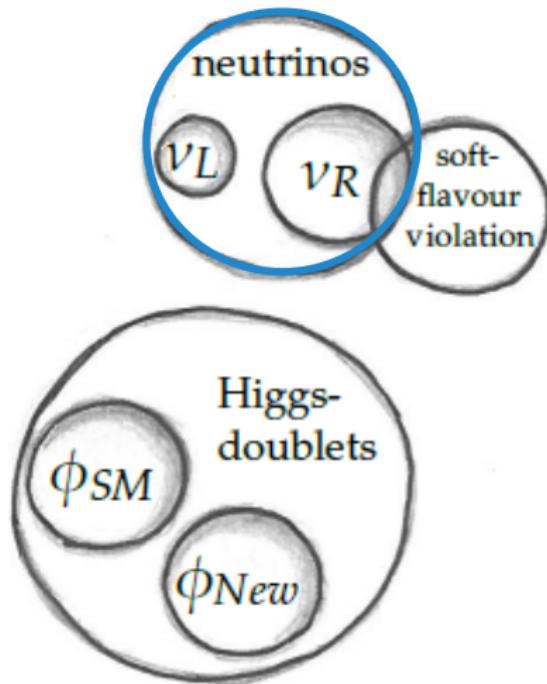
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## CONCLUSION & OUTLOOK

# The model



# The model



# Why we care about neutrinos

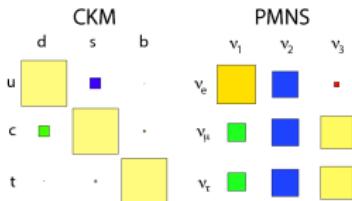
experimentally unsolved: anomalies...

properties:

- just weak interacting
- no observed right handed partner

theoretical unsolved: (all about mass)

- different mixing matrices than quarks
- normal or inverted mass hierarchy
- hierarchy problem: very light mass
- origin of mass: Dirac, Majorana



# Why we care about neutrinos

experimentally unsolved: anomalies...

theoretical unsolved: (all about mass)

- different mixing matrices than quarks
- normal or inverted mass hierarchy
- hierarchy problem: very light mass
- origin of mass: Dirac, Majorana

$$\begin{aligned}\mathcal{L}_M &= ? \quad \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Majorana}} \\ &\sim \bar{\nu}_R M_D \nu_L + \bar{\nu}_\alpha M_M \nu_\beta + h.c.\end{aligned}$$

properties:

- just weak interacting
- no observed right handed partner

Desperately seeking sterile

The three known types of neutrino might be "balanced out" by a bashful fourth type

ELECTRON NEUTRINO	MUON NEUTRINO	TAU NEUTRINO	STERILE NEUTRINO
$\nu_e$	$\nu_\mu$	$\nu_\tau$	$\nu_s$
MASS	< 1 electronvolt	> 1 electronvolt	Gravity
FORCES THEY RESPOND TO	Weak force Gravity	All three "left handed"	"Right handed"
DIRECTION OF SPIN			



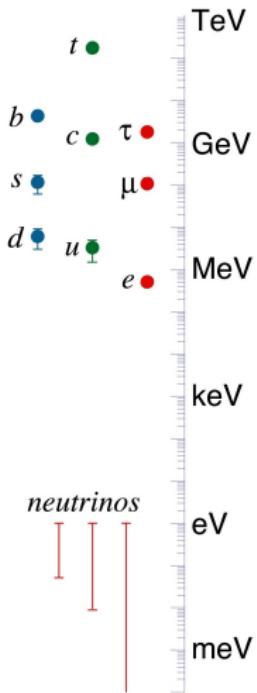
# Heavy right handed neutrinos $\nu_R$

## Hierarchy problem:

Neutrino mass is small  $m_\nu < 0.1 \text{ eV}$  (exp. limits)

Masses are normally  $m_e \simeq 0.5 \text{ MeV}$  to  $m_t \simeq 173 \text{ GeV}$

$\Rightarrow$  *small Yukawa masses seem to be unnatural*



# Heavy right handed neutrinos $\nu_R$

**Soltion:** Majorana neutrinos with heavy  $\nu_R$   
in Seesaw mechanism

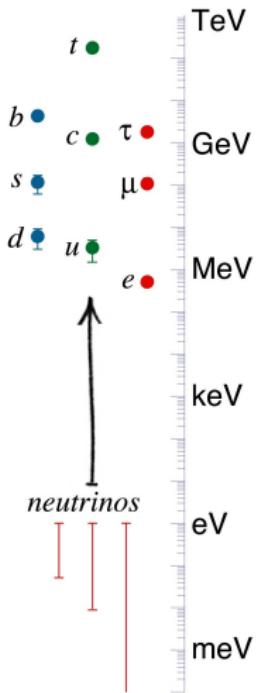
flavour basis: scale  $m_D \sim m_e$ ,  $m_R \gtrsim \text{TeV}$

$$\mathcal{L}_y \supset \mathcal{L}_{\text{dirac}} = \overline{\nu_R} M_D \nu_L + h.c.$$

$$\mathcal{L}_{\text{maj}} = -\frac{1}{2} \overline{(\nu_R)^c} M_R^* \nu_R + h.c.$$

mass basis: diagonalise  $M_{\text{maj}} \rightarrow \text{diag}(\hat{m}_\nu, \hat{m}_R)$

$$\text{scale } m_\nu = -m_D^2/m_R \quad \rightarrow \quad m_\nu \text{ small}$$



# Heavy right handed neutrinos $\nu_R$

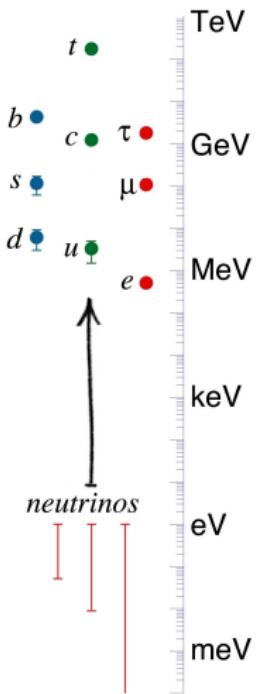
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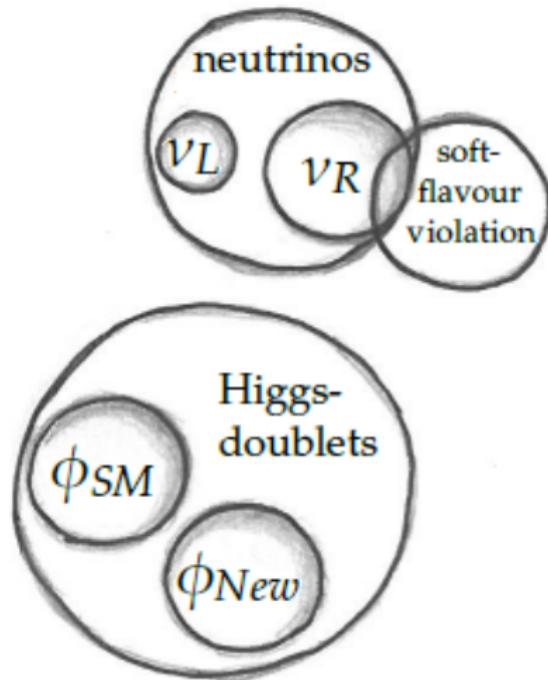
$$\mathcal{L}_{\nu \text{mass}} = -\frac{1}{2} \left( (\overline{\nu_L})^c, \overline{\nu_R} \right) \underbrace{\begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}}_{M_{\text{maj}}} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} + h.c.$$

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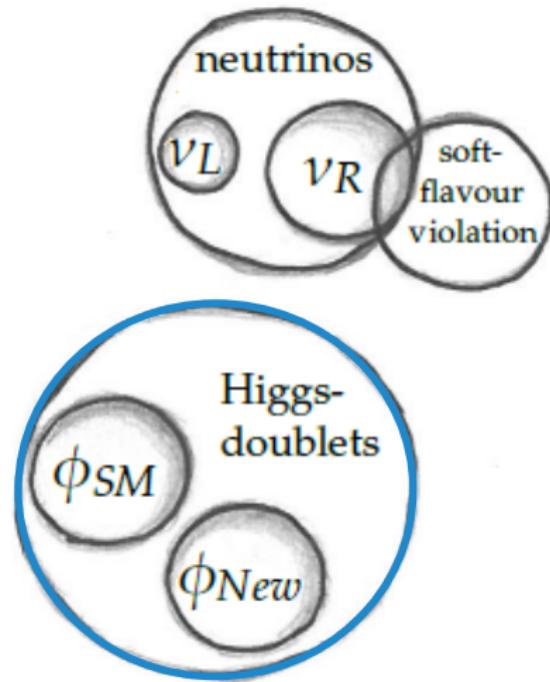
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# The modell



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# Multi Higgs doublets $\Phi_k$

**SM:**  $n_H = 1$  **Higgs doublet**

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \varphi^+ \\ \frac{v}{\sqrt{2}} + \varphi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} + H^0 \end{pmatrix}$$

# Multi Higgs doublets $\Phi_k$

**Include:**  $n_H$  Higgs doublets

$$\phi_{\textcolor{blue}{k}} = \begin{pmatrix} \varphi_{\textcolor{blue}{k}}^+ \\ \varphi_{\textcolor{blue}{k}}^0 \end{pmatrix} = \begin{pmatrix} \varphi_{\textcolor{blue}{k}}^+ \\ \frac{v_{\textcolor{blue}{k}}}{\sqrt{2}} + \varphi_{\textcolor{blue}{k}}^0 \end{pmatrix} = \begin{pmatrix} \sum_a^{n_H} U_{ka} S_a^+ \\ (v_k + \sum_b^{2n_H} V_{kb} S_b^0) / \sqrt{2} \end{pmatrix}$$

**In our case**  $n_H = 2$ :  $\phi_1$  SM Higgs doublet with VEV, and  $\phi_2^{new}$

$$\phi_1 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} + H^0 \end{pmatrix}, \quad \phi_2^{new} = \begin{pmatrix} S^+ \\ (S_3^0 + iS_4^0) / \sqrt{2} \end{pmatrix}$$

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**Include:**  $n_H$  **Higgs doublets**

*Higgs-lepton interaction*

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \quad \varphi^0 = \frac{v}{\sqrt{2}} + \varphi^{0'}$$

$$\mathcal{L}_Y = - \left[ (\varphi^-, \varphi^{0*}) \Gamma \bar{\ell}_R + (\varphi^0, -\varphi^+) \Delta \bar{\nu}_R \right] \begin{pmatrix} \nu'_L \\ \ell'_L \end{pmatrix} + \text{h.c.}$$

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lepton Yukawa couplings

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$$\supset \mathcal{L}_Y(\varphi^\pm) = - \sum_k \left\{ \bar{\ell}_R (\varphi_k^- \Gamma_k) \nu'_L - \bar{\nu}_R (\varphi_k^+ \Delta_k) \ell'_L + h.c. \right\}$$

$$\supset \mathcal{L}_Y(\varphi^0) = - \sum_k \left\{ \bar{\ell}_R \left( \varphi_k^{0'*} \Gamma_k + \underbrace{M_\ell}_{\Gamma_k v_k / \sqrt{2}} \right) \ell'_L + \bar{\nu}_R \left( \varphi_k^{0'} \Delta_k + \underbrace{M_D}_{\Delta_k v_k / \sqrt{2}} \right) \nu'_L + h.c. \right\}$$

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in our case:

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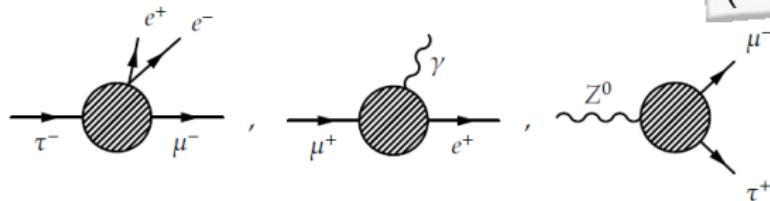
$$-\mathcal{L}_Y(\varphi^0) = \bar{\ell}_R \underbrace{\ell'_L}_{\Gamma_1 v / \sqrt{2}} + \bar{\nu}_R \underbrace{\nu'_L}_{\Delta_1 v / \sqrt{2}} + \bar{\ell}_R H^0 \Gamma_1 \ell'_L + \bar{\ell}_R (\varphi_3^0 + i \varphi_4^0) \Gamma_2 \ell'_L + \dots$$

# Multi Higgs doublets $\Phi_k$

**Include:**  $n_H$  **Higgs doublets**

Higgs-lepton interaction

e.g.



interesting effect:  
**observable processes!**  
(Flavour Changing Neutral Int.)

in our case:

$$-\mathcal{L}_Y(\varphi^\pm) = \bar{\ell}_R(\varphi^- \Gamma_2) \nu'_L - \bar{\nu}_R(\varphi^+ \Delta_2) \ell'_L + h.c.$$

flav. non-diag.

$$-\mathcal{L}_Y(\varphi^0) = \bar{\ell}_R \underbrace{\frac{M_\ell}{\Gamma_1 v / \sqrt{2}}}_{\ell'_L} + \bar{\nu}_R \underbrace{\frac{M_D}{\Delta_1 v / \sqrt{2}}}_{\nu'_L} + \bar{\ell}_R H^0 \Gamma_1 \ell'_L + \bar{\ell}_R (\varphi_3^0 + i \varphi_4^0) \Gamma_2 \ell'_L + \dots$$

# Multi Higgs doublets $\Phi_k$

## Problem:

*strong experimental bounds on FCNIs at tree level!*

e.g.

$$\begin{array}{c} \text{---} \\ \tau \quad \mu \end{array} \circledast = \begin{array}{c} \text{---} \\ \tau \quad \mu \end{array} S^0 + \text{one-loop} \\
 \sim |\Gamma_2^{\tau\mu} \Gamma_2^{ee} / m_{S^0}^2| \\
 \text{suppressed}
 \end{array}$$

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tree-level

# Multi Higgs doublets $\Phi_k$

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$S^0$

$\sim |\Gamma_2^{\tau\mu} \Gamma_2^{ee} / m_{S^0}^2|$   
*suppressed*

solution:

either very small  $\Gamma_2$  & big  $m_{S^0}$

or Lepton flavour symmetry  
flavour diag.  $\Gamma, \Delta$

$$-\mathcal{L}_Y(\varphi^\pm) = \bar{\ell}_R(\varphi^- \Gamma_2) \nu'_L - \bar{\nu}_R(\varphi^+ \Delta_2) \ell'_L + h.c.$$

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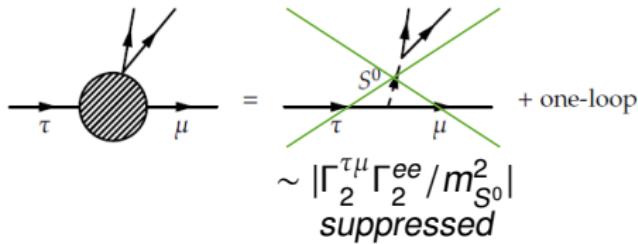
tree-level

# Multi Higgs doublets $\Phi_k$

## Problem:

*strong experimental bounds on FCNIs at tree level!*

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solution:

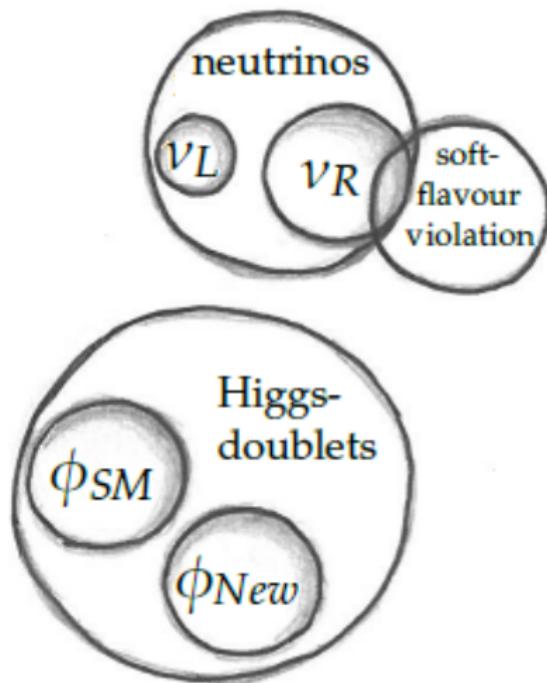
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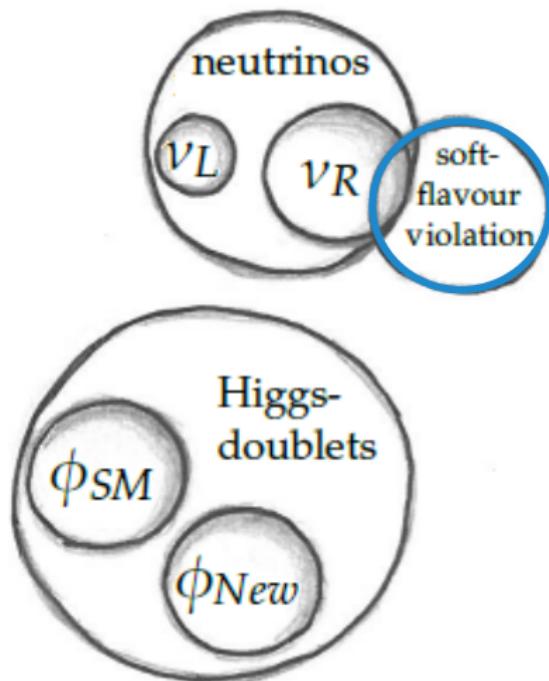
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tree-level

# The modell



# The modell



# Lepton flavour

**SM:**

$$\begin{aligned} -\mathcal{L}_Y(\varphi^0) &= \bar{\ell}_R \underbrace{M_\ell}_{\Gamma_1 v/\sqrt{2}} \ell'_L + \\ &\quad \bar{\ell}_R H^0 \Gamma_1 \ell'_L + \dots \\ &\quad \text{↑} \\ &\quad \ell\text{-mass basis} \\ &\quad \downarrow \\ -\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma^\mu W_\mu^+ \nu_L + h.c. \end{aligned}$$

# Lepton flavour

**SM + mixing neutrinos:**

$$-\mathcal{L}_Y(\varphi^0) = \bar{\ell}_R \underbrace{M_\ell}_{\Gamma_1 v/\sqrt{2}} \ell'_L + \bar{\nu}_R \underbrace{M_D}_{\Delta_1 v/\sqrt{2}} \nu'_L + \bar{\ell}_R H^0 \Gamma_1 \ell'_L + \dots$$

$\uparrow$   
 $\ell$ -mass basis  $\neq$   $\nu$ -mass basis  
 $\downarrow$

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma^\mu \underbrace{U_{PMNS}}_{U_\ell^\dagger U_\nu} W_\mu^+ \nu_L + h.c.$$

$$= \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma^\mu W_\mu^+ \hat{\nu}_L + h.c$$

flavour basis

# Lepton flavour

**SM + mixing neutrinos +  $\Phi_2$ :**

$$\begin{aligned}
 -\mathcal{L}_Y(\varphi^0) &= \bar{\ell}_R \underbrace{M_\ell}_{\Gamma_1 v/\sqrt{2}} \ell'_L + \bar{\nu}_R \underbrace{M_D}_{\Delta_1 v/\sqrt{2}} \nu'_L + \bar{\ell}_R H^0 \Gamma_1 \ell'_L + \bar{\ell}_R (\varphi_3^0 + i\varphi_4^0) \Gamma_2 \ell'_L + \dots \\
 &\quad \text{non-diag.} \\
 -\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma^\mu \underbrace{U_{PMNS}}_{U_\ell^\dagger U_\nu} W_\mu^+ \nu_L + h.c. \\
 &= \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma^\mu W_\mu^+ \hat{\nu}_L + h.c. \\
 &\quad \text{flavour basis}
 \end{aligned}$$

$\uparrow$                              $\uparrow$   
 $\ell$ -mass basis  $\neq$   $\nu$ -mass basis  
 $\downarrow$                              $\downarrow$

# Lepton flavour symmetry: $L_\alpha$

$U(1)_e \times U(1)_\mu \times U(1)_\tau$  continuous, global

Symmetry transf.

$$U_e(\theta) = \begin{pmatrix} e^{iL_e\theta} & & \\ & 1 & \\ & & 1 \end{pmatrix}, \quad U_\mu(\theta) = \begin{pmatrix} 1 & & \\ & e^{iL_\mu\theta} & \\ & & 1 \end{pmatrix}, \quad U_\tau(\theta) = \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{iL_\tau\theta} \end{pmatrix}$$

Fermion transf.

$$\binom{\nu_\ell}{\ell} \rightarrow U_\alpha(\theta) \binom{\nu_\ell}{\ell}, \quad \ell_R \rightarrow U_\alpha(\theta) \ell_R, \quad \nu_R \rightarrow U_\alpha(\theta) \nu_R$$

$\Rightarrow \underline{\mathcal{L} \supset \mathcal{L}_Y, \mathcal{L}_{CC}}$  invariant under  $L_\alpha$ -symmetry

# Lepton flavour symmetry

**SM + mixing neutrinos +  $\Phi_2$ :**

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diag.!

$$\begin{array}{ccc} \Gamma_1 v/\sqrt{2} & & \Delta_1 v/\sqrt{2} \\ \uparrow & & \uparrow \\ \ell\text{-mass basis} & = & \nu\text{-mass basis} \\ \downarrow & & \downarrow \end{array}$$

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- ⇒ Yukawa couplings  $\Gamma, \Delta$  diag.
- ⇒ no tree-level mixing ✓
- ⇒ no  $\nu$ -mixing:  $U_{PMNS} = \mathbb{1}$  ↴

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# Lepton flavour symmetry

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- break  $L_\alpha$  soft

⇒  $\mathcal{L} \supset \mathcal{L}_Y, \mathcal{L}_{CC}$  invariant under  $L_\alpha$ -symmetry

# Soft flavour breaking $L_\alpha$

Impose: Lepton flavour symmetry

$$\begin{aligned}-\mathcal{L}_Y(\varphi^0) &= \bar{\ell}_R \textcolor{brown}{M}_\ell \ell'_L + \bar{\nu}_R \textcolor{brown}{M}_D \nu'_L + \bar{\ell}_R H^0 \Gamma_1 \ell'_L + \bar{\ell}_R (\varphi_3^0 + i\varphi_4^0) \Gamma_2 \ell'_L + \dots \\ -\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma_\mu \underbrace{U_\ell^\dagger U_\nu}_{\mathbb{1}} W^{+\mu} \nu_L + h.c.\end{aligned}$$

Break: Lepton flavour soft

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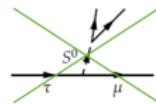
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 \end{aligned}$$

**Break:** Lepton flavour soft

explicit breaking in  $O(3)$  terms in energy dim. of fields (all terms  $\leq O(3)$  must be present)

- ⇒  $O(3)$ : two fermions involved, propagator- or mass-term
- ⇒ no flavour-violating vertex, no FCNIs at tree level! ✓
- ⇒ no counterterms from a vertex → no UV divergences at one loop
- ⇒ one loop contributions finite!



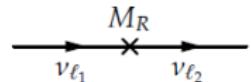
# Soft flavour breaking $L_\alpha$

Impose: Lepton flavour symmetry

$$\begin{aligned}
 -\mathcal{L}_Y(\varphi^0) &= \bar{\ell}_R \textcolor{brown}{M}_\ell \ell'_L + \bar{\nu}_R \textcolor{brown}{M}_D \nu'_L + \bar{\ell}_R H^0 \Gamma_1 \ell'_L + \bar{\ell}_R (\varphi_3^0 + i\varphi_4^0) \Gamma_2 \ell'_L + \dots \\
 -\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma_\mu \underbrace{U_{PMNS}}_{U_\nu \neq \mathbb{1}} W^{+\mu} \nu_L + h.c.
 \end{aligned}$$

**Break:** Lepton flavour soft

$$\mathcal{L}_{maj} = -\frac{1}{2} \overline{(\nu_R)^c} \underbrace{\textcolor{brown}{M}_R^*}_{non-diag.} \nu_R + h.c.$$



$$\begin{aligned}
 \mathcal{L}_{v mass} &= -\frac{1}{2} \left( \overline{(\nu_L)^c}, \overline{\nu_R} \right) \begin{pmatrix} 0 & M_D^T \\ M_D & \textcolor{brown}{M}_R \end{pmatrix} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} + h.c. && \text{flavour basis} \\
 &= -\frac{1}{2} \bar{\chi} U^T (\textcolor{brown}{U}_{PMNS}) \begin{pmatrix} \hat{m}_L & 0 \\ 0 & \hat{m}_R \end{pmatrix} U (\textcolor{brown}{U}_{PMNS}) \chi + h.c. && \text{mass basis}
 \end{aligned}$$

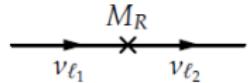
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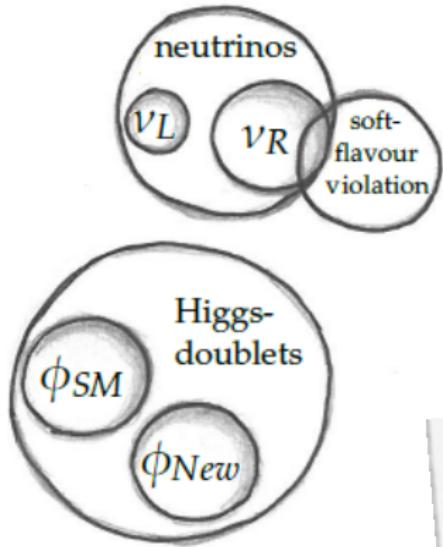
$$\mathcal{L}_{maj} = -\frac{1}{2} \overline{(\nu_R)^c} \underbrace{M_R^*}_{non-diag.} \nu_R + h.c.$$



- $\nu_R$  responsible for flavour mixing ( $U_{PMNS}$ )
- FCNIs at one loop in  $\ell$ -decays

# Properties and advantages

- explain tiny  $\nu$ -masses
- additional particles:  $\varphi_3, \varphi_4, \nu_R$
- elegant symmetry:
  - no FCNI at tree-level
  - one-loop contributions are finite
  - explains  $U_{PMNS}$  &  $V_{CKM}$  different

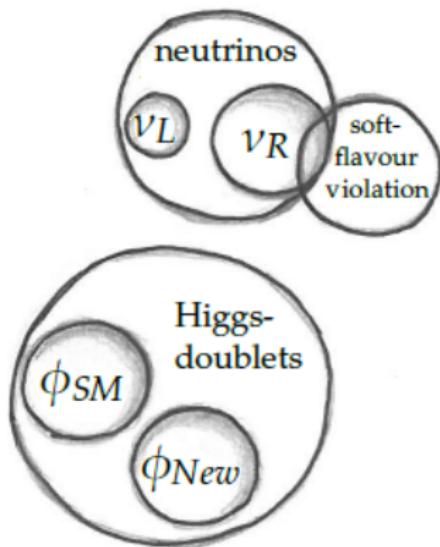


modell generates:  
atm. & sol. maximal mixing  
[Grimus, 01]

PMNS		
$\nu_e$	$\nu_1$	$\nu_2$
$\nu_\mu$	■	■
$\nu_\tau$	■	■

# Properties and advantages

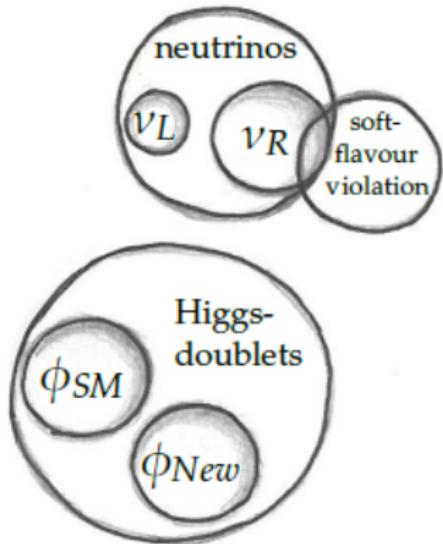
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## open questions:

- model experimental testable in FCNIs
- solves the exp.-theor. discrepancy of  $\mu$ -MDM?

# Properties and advantages



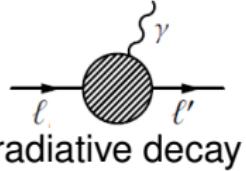
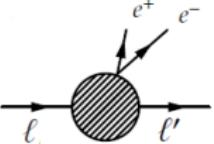
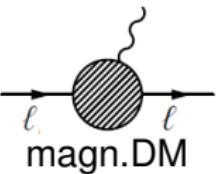
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## Test with Charged lepton decays

# Charged lepton decays

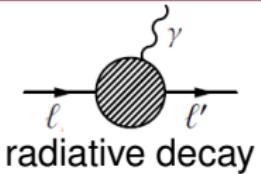
high exp. bounds	model contributions
$\text{BR}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13}$ $\text{BR}(\tau^- \rightarrow e^- \gamma) < 3.3 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow \mu^- \gamma) < 4.4 \times 10^{-8}$	 <b>radiative decay</b>
$\text{BR}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}$ $\text{BR}(\tau^- \rightarrow e^- e^+ e^-) < 2.7 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow e^- \mu^+ \mu^-) < 2.7 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 2.1 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow \mu^- e^+ e^-) < 1.8 \times 10^{-8}$	 <b>three part. decay</b>
$a_e^{\text{exp}-\text{err}} = \pm 2.6 \times 10^{-13} \quad (\text{exp/SM} = 1)$ $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (287 \pm 80) \times 10^{-11} \\ (261 \pm 78) \times 10^{-11} \end{cases} \quad (3.6\sigma)$	 <b>magn.DM</b>

# Charged lepton decays

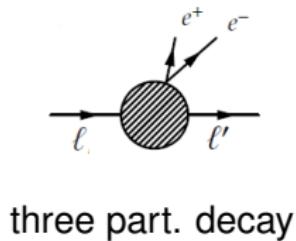
high exp. bounds

$$\begin{aligned} \text{BR}(\mu^+ \rightarrow e^+ \gamma) &< 4.2 \times 10^{-13} \\ \text{BR}(\tau^- \rightarrow e^- \gamma) &< 3.3 \times 10^{-8} \\ \text{BR}(\tau^- \rightarrow \mu^- \gamma) &< 4.4 \times 10^{-8} \end{aligned}$$

model contributions

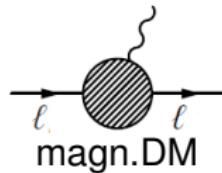


$$\begin{aligned} \text{BR}(\mu^- \rightarrow e^- e^+ e^-) &< 1.0 \times 10^{-12} \\ \text{BR}(\tau^- \rightarrow e^- e^+ e^-) &< 2.7 \times 10^{-8} \\ \text{BR}(\tau^- \rightarrow e^- \mu^+ \mu^-) &< 2.7 \times 10^{-8} \\ \text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) &< 2.1 \times 10^{-8} \\ \text{BR}(\tau^- \rightarrow \mu^- e^+ e^-) &< 1.8 \times 10^{-8} \end{aligned}$$



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# Short analysis of $\ell$ -decays

## One-loop model contributions:

$$\mathcal{A}(Z \rightarrow \tau^+ \mu^-) \propto 1/m_R^2,$$

$$\mathcal{A}(\tau^- \rightarrow \mu^- \gamma) \propto 1/m_R^2,$$

$$\mathcal{A}(\tau^- \rightarrow \mu^- e^+ e^-) \propto \begin{cases} 1/m_R^2 & n_H = 1 \\ \text{const.} & n_H > 1 \end{cases}$$

*characteristics  
of this model*

Processes including the sub-process  $\ell^- \rightarrow \ell'^- S^{0*}$ , ( $S^{0*} \rightarrow e^+ e^-$ ) have ( $n_H \geq 2$ ) non- $m_R$ -suppressed contributions from graphs with charged-scalar exchange  $S^\pm$  (plot) in their Amplitudes  $\mathcal{A}$ , [Grimus, Lavoura, 02].

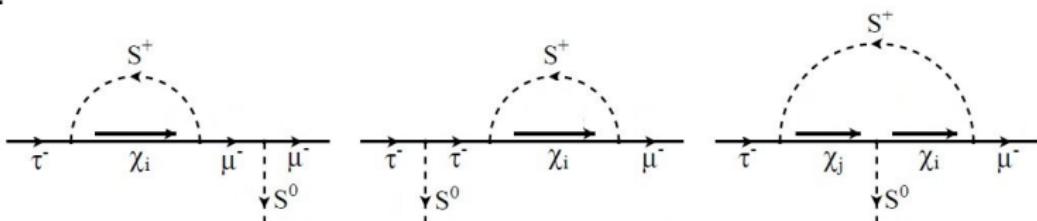


Figure: non-suppressed diagrams for  $\tau^- \rightarrow \mu^- S^{0*}$

# Short analysis of $\ell$ -decays

## One-loop model contributions:

*characteristics  
of this model*

$$\mathcal{A}(Z \rightarrow \tau^+ \mu^-) \propto 1/m_R^2, \text{ suppression}$$

$$\mathcal{A}(\tau^- \rightarrow \mu^- \gamma) \propto 1/m_R^2, \text{ via } m_R$$

$$\mathcal{A}(\tau^- \rightarrow \mu^- e^+ e^-) \propto \begin{cases} 1/m_R^2 & n_H = 1 \\ \text{const.} & n_H > 1 \end{cases} \text{ only contr.}$$

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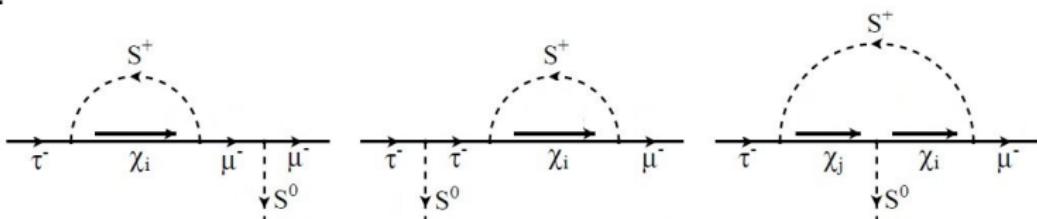
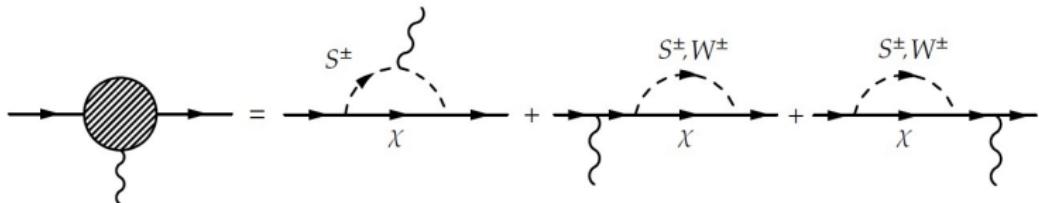


Figure: non-suppressed diagrams for  $\tau^- \rightarrow \mu^- S^{0*}$

# Suppression of radiative corrections



$$\Gamma(\ell_1^\pm \rightarrow \ell_2^\pm \gamma) = \frac{\alpha m_{\ell_1}^3}{4} (|A_L|^2 + |A_R|^2), \quad \text{with } A_{L,R} \sim \frac{1}{16\pi^2} \frac{m_{\ell_1}}{m_R^2}$$

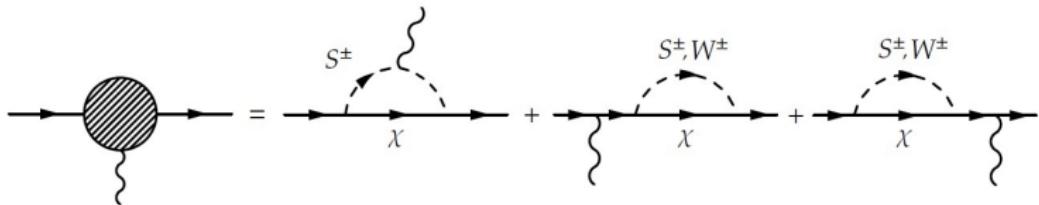
A lower bound on  $m_R$  the seesaw scale:

$$\text{BR}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13} \quad \Rightarrow \quad m_R \gtrsim 50 \text{ TeV}$$

$$\text{BR}(\tau^- \rightarrow e^- \gamma) < 3.3 \times 10^{-8} \quad \Rightarrow \quad m_R \gtrsim 2 \text{ TeV}$$

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$$\Rightarrow m_R \gtrsim 2 \text{ TeV}$$

$m_R \gtrsim 500 \text{ TeV}$

# Model tested: for $m_R \gtrsim 500 \text{ TeV}$

exp. bounds	model contributions
$\text{BR}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13}$ $\text{BR}(\tau^- \rightarrow e^- \gamma) < 3.3 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow \mu^- \gamma) < 4.4 \times 10^{-8}$	$\sim \frac{1}{m_R^4}$ <b>suppressed!</b>
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$a_e^{\text{exp-err}} = \pm 2.6 \times 10^{-13} \quad (\text{exp/SM} = 1)$	<b>solved?</b>
$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (287 \pm 80) \times 10^{-11} \\ (261 \pm 78) \times 10^{-11} \end{cases} \quad (3.6\sigma)$	

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# Free Parameter

**seesaw scale:**  $\hat{m}_R = \begin{pmatrix} m_4 & & \\ & m_5 & \\ & & m_6 \end{pmatrix} \gtrsim \mathcal{O}(500 \text{ TeV})$

**Yukawa couplings:**

$$\left( \Gamma_1 = \frac{\sqrt{2}}{v} \text{diag}(m_e, m_\mu, m_\tau) = \frac{\sqrt{2}}{v} M_\ell \right)$$

$$\Gamma_2 = \text{diag}(\gamma_e, \gamma_\mu, \gamma_\tau)$$

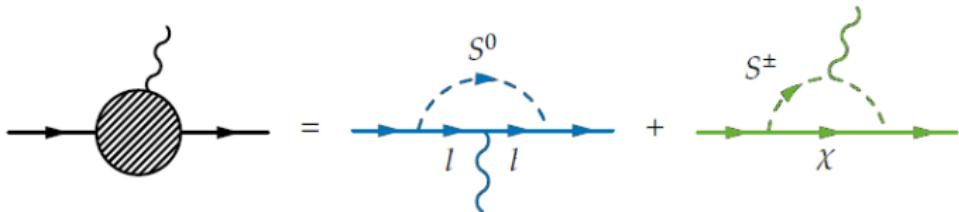
$$\Delta_1 = \text{diag}(d_e, d_\mu, d_\tau) = \frac{\sqrt{2}}{v} M_D$$

$$\Delta_2 = \text{diag}(\delta_e, \delta_\mu, \delta_\tau)$$

**scalar masses:** SM Higgs boson, 2 other scalars:

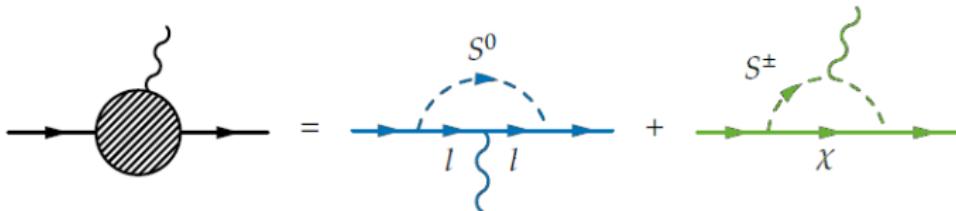
$$M_3, M_4, (M_{\text{higgs}} = 125 \text{ TeV})$$

# Magnetic dipole moments



$$\begin{aligned}
 a_\ell^{(S)} \simeq & \frac{m_\ell^2}{96\pi^2} \left\{ 2|\gamma_\ell|^2 \left( \frac{1}{M_3^2} + \frac{1}{M_4^2} \right) \right. \\
 & - 3 \operatorname{Re} \left( e^{2i\alpha} \gamma_\ell^2 \right) \left[ \frac{1}{M_3^2} \left( 3 + 2 \ln \frac{m_\ell^2}{M_3^2} \right) - \frac{1}{M_4^2} \left( 3 + 2 \ln \frac{m_\ell^2}{M_4^2} \right) \right] \\
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 \end{aligned}$$

free parameters:  $M_3, M_4, \gamma_\mu, \gamma_e$

# Magnetic dipole moments

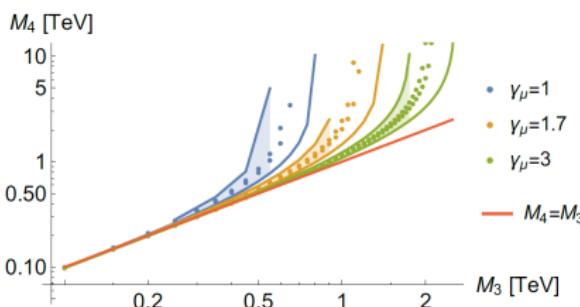
**Constrains on  $M_3, M_4, \gamma_\mu, \gamma_e$ :**

$\mu$ -MDM

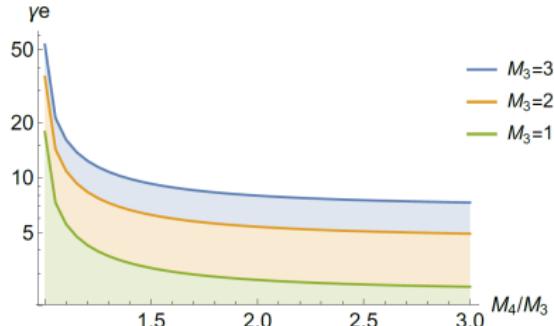
$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (287 \pm 80) \times 10^{-11} (\text{at } 3.6\sigma)^a \\ (261 \pm 78) \times 10^{-11} (\text{at } 3.6\sigma)^b \end{cases}$$

$e$ -MDM

$$a_e^{\text{err}} = \pm 2.6 \times 10^{-13}$$



$$\begin{aligned} M_3, M_4 &\gtrsim 1 \text{ TeV} \\ \Rightarrow \quad \gamma_\mu &\geq 1.7 \end{aligned}$$



$$\begin{aligned} M_3 &= 1 \text{ TeV}, M_4 = 2 \text{ TeV} \\ \Rightarrow \quad \gamma_e &< 2.76 \end{aligned}$$

<sup>a</sup>[1010.4180/hep-ph], <sup>b</sup>[1105.3149/hep-ph]

## Further assumptions for simplicity:

- all parameters are real, also  $U_{PMNS}$
- additional phases:  $e^{i\hat{\alpha}} = e^{i\hat{\beta}} = 1$

**BR( $\ell \rightarrow 3\ell$ ) close to exp. bounds...**

# Parameter set

**seesaw scale:**

$$\hat{m}_R = \begin{pmatrix} m_4 & & \\ & m_5 & \\ & & m_6 \end{pmatrix} \gtrsim \mathcal{O}(500 \text{ TeV})$$

**Yukawa couplings:**

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$$\Gamma_2 = \text{diag}(\gamma_e, \gamma_\mu, \gamma_\tau) = \text{diag}(1.7, 1.7, 1.7)$$

$$\Delta_1 = \text{diag}(d_e, d_\mu, d_\tau) = \text{diag}(0.6, 0.1, 0.1)$$

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$$M_3 = 1 \text{ TeV}, M_4 = 2 \text{ TeV}, (M_{higgs} = 125 \text{ GeV})$$

[EA, Grimus, Lavoura, 17]

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$\Gamma, \Delta$  not small       $\Delta_1 = \text{diag}(d_e, d_\mu, d_\tau) = \text{diag}(0.6, 0.1, 0.1)$

$$\Delta_2 = \text{diag}(\delta_e, \delta_\mu, \delta_\tau) = \text{diag}(0, 0.00007, 0.2)$$

**scalar masses:** SM Higgs boson, Goldstone boson, 2 other scalars:

not too large  $M_3 = 1 \text{ TeV}$ ,  $M_4 = 2 \text{ TeV}$ , ( $M_{higgs} = 125 \text{ GeV}$ )

[EA, Grimus, Lavoura, 17]

# Results

exp. bounds		model contributions
$\text{BR}(\mu^+ \rightarrow e^+ \gamma)$	< $4.2 \times 10^{-13}$	
$\text{BR}(\tau^- \rightarrow e^- \gamma)$	< $3.3 \times 10^{-8}$	$\sim \frac{1}{m_R^4}$
$\text{BR}(\tau^- \rightarrow \mu^- \gamma)$	< $4.4 \times 10^{-8}$	
$\text{BR}(\mu^- \rightarrow e^- e^+ e^-)$	< $1.0 \times 10^{-12}$	$3.872 \times 10^{-13}$
$\text{BR}(\tau^- \rightarrow e^- e^+ e^-)$	< $2.7 \times 10^{-8}$	$1.111 \times 10^{-8}$
$\text{BR}(\tau^- \rightarrow e^- \mu^+ \mu^-)$	< $2.7 \times 10^{-8}$	$1.280 \times 10^{-8}$
$\text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-)$	< $2.1 \times 10^{-8}$	$1.307 \times 10^{-8}$
$\text{BR}(\tau^- \rightarrow \mu^- e^+ e^-)$	< $1.8 \times 10^{-8}$	$1.506 \times 10^{-8}$
$a_e^{\text{exp-err}} = \pm 2.6 \times 10^{-13}$ ( $\text{exp/SM} = 1$ )		$a_e^{\text{mod}} = 1.0 \times 10^{-13}$
$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (287 \pm 80) \times 10^{-11} \\ (261 \pm 78) \times 10^{-11} \end{cases}$ ( $3.6\sigma$ )		$a_\mu^{\text{mod}} = 258 \times 10^{-11}$

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$\text{BR}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}$ $\text{BR}(\tau^- \rightarrow e^- e^+ e^-) < 2.7 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow e^- \mu^+ \mu^-) < 2.7 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 2.1 \times 10^{-8}$ $\text{BR}(\tau^- \rightarrow \mu^- e^+ e^-) < 1.8 \times 10^{-8}$	<i>Mu3e, impr.</i> $10^{-16}$	$3.872 \times 10^{-13}$ $1.111 \times 10^{-8}$ $1.280 \times 10^{-8}$ $1.307 \times 10^{-8}$ $1.506 \times 10^{-8}$
$a_e^{\text{exp-err}} = \pm 2.6 \times 10^{-13}$ ( $\text{exp/SM} = 1$ )		$a_e^{\text{mod}} = 1.0 \times 10^{-13}$
$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (287 \pm 80) \times 10^{-11} \\ (261 \pm 78) \times 10^{-11} \end{cases}$ ( $3.6\sigma$ )		$a_\mu^{\text{mod}} = 258 \times 10^{-11}$
<b>and other experiments...</b>		

# Conclusion

## Model:

- explains light  $m_\nu$
- introduce new particles:  $\varphi_3, \varphi_4, \nu_R^{maj}$   
without heavy restr. on  $\Gamma, \Delta, m_S$  from FCNI (broken L-symmetry)
- explains difference:  $U_{PMNS}, V_{CKM}$   
 $\Rightarrow$  lepton flavour violation comes solely from  $\nu_R$

## $\ell$ -decays:

- Explains exp. and theo. discrepancy of  $\mu$ -MDM
- Pointing out experimental signatures

# Outlook

- examine  $m_R < 500$  TeV, look at suppressed diag.  
[Talk: D. Jurciukonis yesterday]
- examine corrections to  $\nu$ -mass
- get more boundaries on our parameters  
for more information about  $\Gamma$ ,  $\Delta$
- searching for compatibility with other models/problems

# Outlook

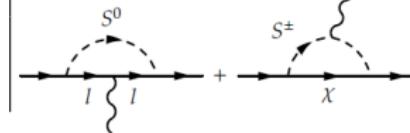
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**Thank you!**

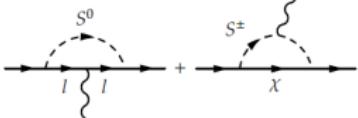
# Backup slides



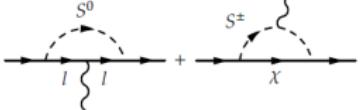
# Charged lepton decays

exp. bounds	model contributions
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$(\text{exp/SM} = 1) \quad a_e^{err} = \pm 2.6 \times 10^{-13}$ $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (287 \pm 80) \times 10^{-11} \\ (261 \pm 78) \times 10^{-11} \end{cases}$	$a_\ell \subset \mathcal{A}(\ell \rightarrow \ell \gamma) \sim$ 

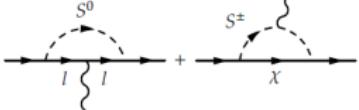
# Charged lepton decays

exp. bounds	model contributions	come out
$\text{BR}(\mu^+ \rightarrow e^+ \gamma)$ $\text{BR}(\tau^- \rightarrow e^- \gamma)$ $\text{BR}(\tau^- \rightarrow \mu^- \gamma)$	$\mathcal{A}(\tau^- \rightarrow \mu^- \gamma) \propto \frac{1}{m_R^2}$	
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$a_e^{\text{err}}$ $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	$a_\ell \subset \mathcal{A}(\ell \rightarrow \ell \gamma) \sim$ 	

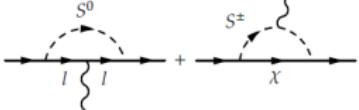
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$a_e^{err}$ $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	$a_\ell \subset \mathcal{A}(\ell \rightarrow \ell \gamma) \sim$ 	

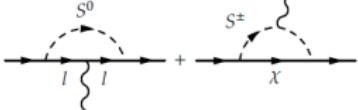
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$a_e^{err}$ $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	$a_\ell \subset \mathcal{A}(\ell \rightarrow \ell \gamma) \sim$ 	<b>explain <math>\mu</math>-MDM</b>

# Charged lepton decays

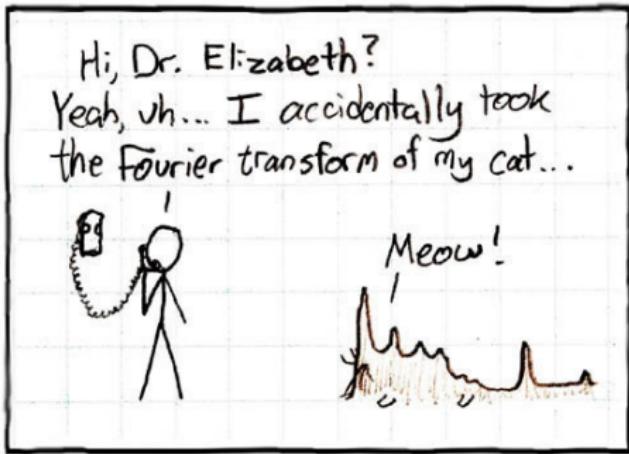
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$a_e^{\text{err}}$ $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	$a_\ell \subset \mathcal{A}(\ell \rightarrow \ell \gamma) \sim$ 	<b>explain <math>\mu</math>-MDM</b> → constrain free param.

# Charged lepton decays

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Things might become a bit technical here...

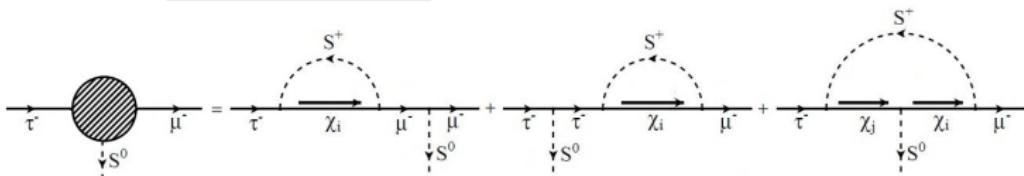
calculating problems:



# Finding free parameters

**Goal:  $\text{BR}(\ell_1^- \rightarrow \ell_2^- \ell_3^+ \ell_3^-)$  close to exp. bounds**

→ constrain **free parameters** in non-suppressed contributions to BR:



BR calculation (one loop):

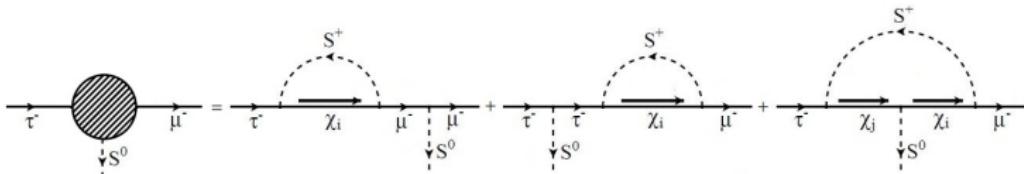
( $S^0$ : Higgs  $\varphi^0$  in mass space)

$$\mathcal{L}_Y(S^0) = -\frac{1}{\sqrt{2}} \sum_{b\ell} S_b^0 \bar{\ell} \left[ (\hat{\Gamma}_b)_{\ell\ell} \gamma_L + (\hat{\Gamma}_b^\dagger)_{\ell\ell} \gamma_R \right] \ell$$

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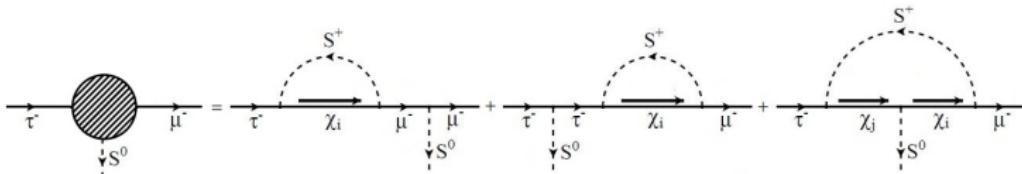
( $S^0$ : Higgs  $\varphi^0$  in mass space)

$$\mathcal{L}_{Y\text{eff}}(S^0) = \sum_{b, \ell_1 \neq \ell_2} S^0 \bar{\ell}_1 \left[ (A_L^b)_{\ell_1 \ell_2} \gamma_L + (A_R^b)_{\ell_1 \ell_2} \gamma_R \right] \ell_2 \text{ (one loop)}$$

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$$\rightarrow \Gamma(\ell_1^- \rightarrow \ell_2^- \ell_3^+ \ell_3^-) = \frac{m_{\ell_1}^5}{6144\pi^3} |X_{\ell_2 \ell_1}|^2 |\gamma_{\ell_3}|^2 \frac{|A_{\ell_2 \ell_1}|^2 + |A_{\ell_1 \ell_2}|^2}{(m_{\ell_2}^2 - m_{\ell_1}^2)^2} \left( \frac{1}{M_3^4} + \frac{1}{M_4^4} \right)$$

$$\text{BR}(\ell) = \Gamma(\ell)/\Gamma_{\text{tot}}$$

# Free parameters

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$$X_{\ell_1 \ell_2} \sim \sum_{i=4}^6 (U_R)_{\ell_1 i} (U_R^*)_{\ell_2 i} \ln \frac{m_i^2}{\mu^2},$$

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$$M_3, M_4, (M_{\text{higgs}} = 125 \text{ GeV})$$

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## Seesaw scale - Dirac masses

With the choice for neutrino mass  $m_1 = 0.05 \text{ eV}$ ,  
 given best-fit values  $\Delta m_{21}^2 = 7.50 \times 10^{-5} \text{ eV}^2$ ,  $\Delta m_{31}^2 = 2.457 \times 10^{-3} \text{ eV}^2$   
 $(m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, m_3 = \sqrt{m_1^2 + \Delta m_{31}^2})$   
 and fixing  $\Delta_1 \sim d_\ell$  we can calculate  $M_R = \text{diag}(m_4, m_5, m_6) \sim O(m_R)$ :

$$M_\nu = -M_D^T M_R^{-1} M_D, \quad M_D = -\frac{v}{\sqrt{2}} \Delta_1$$

$$\begin{aligned} M_R &= -\frac{v^2}{2} \Delta_1 M_\nu^{-1} \Delta_1 \\ &= -\frac{v^2}{2} e^{i\hat{\alpha}} \Delta_1 U_{\text{PMNS}} (e^{2i\hat{\beta}} \hat{m}^{-1}) U_{\text{PMNS}}^T \Delta_1 e^{i\hat{\alpha}} \end{aligned}$$

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$$\begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix} \xrightarrow{2 \times 2 \text{ diag.}} \begin{pmatrix} M_\nu & \\ & M_R \end{pmatrix} \xrightarrow{\text{diag. } (U_{\text{PMNS}})} \begin{pmatrix} \hat{m}_\nu & \\ & \hat{m}_R \end{pmatrix}$$

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- **Dirac masses** ( $m_{D\ell} = \frac{v}{\sqrt{2}} d_\ell$ ):  $d_e = 0.6, d_\mu = d_\tau = 0.1$
- **Seesaw scale**  $m_R > 10^{12} \text{ GeV}$ :

$$m_4 = 4.3 \times 10^{12} \text{ GeV}, \quad m_5 = 6.0 \times 10^{12} \text{ GeV}, \quad m_6 = 2.2 \times 10^{14} \text{ GeV}$$

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 and fixing  $\Delta_1 \sim d_\ell$  we can calculate  $M_R = \text{diag}(m_4, m_5, m_6) \sim O(m_R)$ :

$$M_\nu = -M_D^T M_R^{-1} M_D, \quad M_D = -\frac{v}{\sqrt{2}} \Delta_1$$

$$\begin{aligned} M_R &= -\frac{v^2}{2} \Delta_1 M_\nu^{-1} \Delta_1 \\ &= -\frac{v^2}{2} e^{i\hat{\alpha}} \Delta_1 U_{\text{PMNS}} (e^{2i\hat{\beta}} \hat{m}^{-1}) U_{\text{PMNS}}^T \Delta_1 e^{i\hat{\alpha}} \end{aligned}$$

- **Dirac masses** ( $m_{D\ell} = \frac{v}{\sqrt{2}} d_\ell$ ):  $d_e = 0.6, d_\mu = d_\tau = 0.1$
- **Seesaw scale**  $m_R > 10^{12} \text{ GeV}$ : ✓ rad. decay (> 500 TeV)  
 $m_4 = 4.3 \times 10^{12} \text{ GeV}, m_5 = 6.0 \times 10^{12} \text{ GeV}, m_6 = 2.2 \times 10^{14} \text{ GeV}$

# Free parameters

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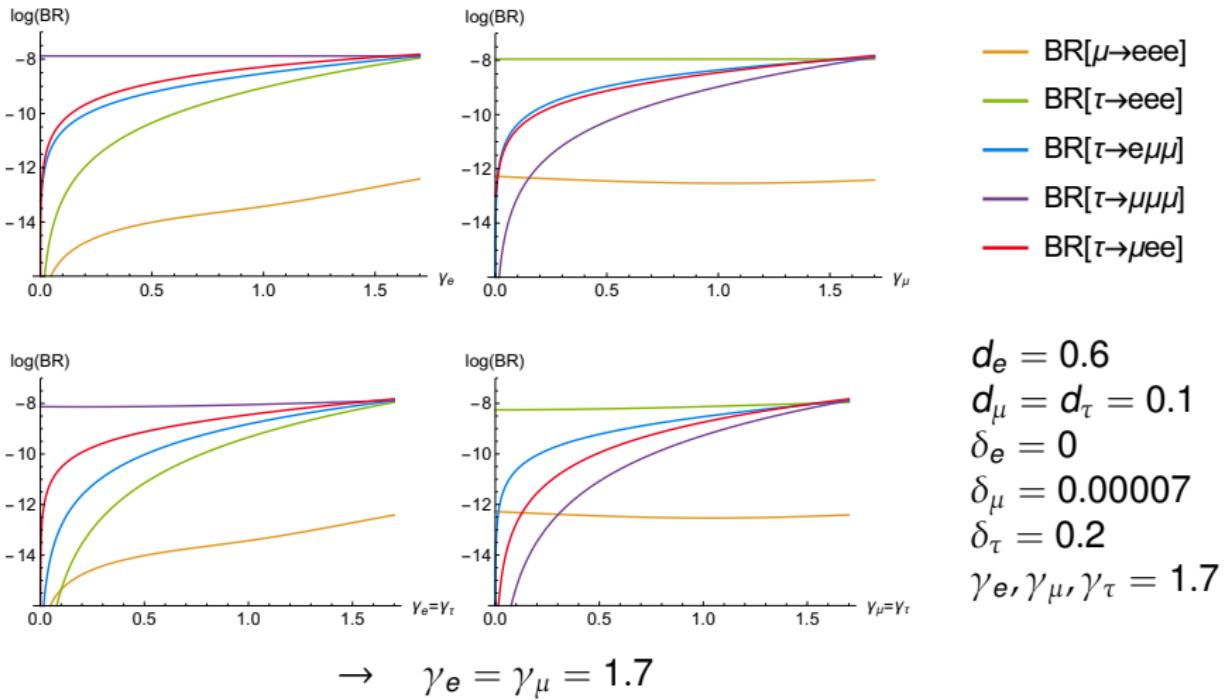
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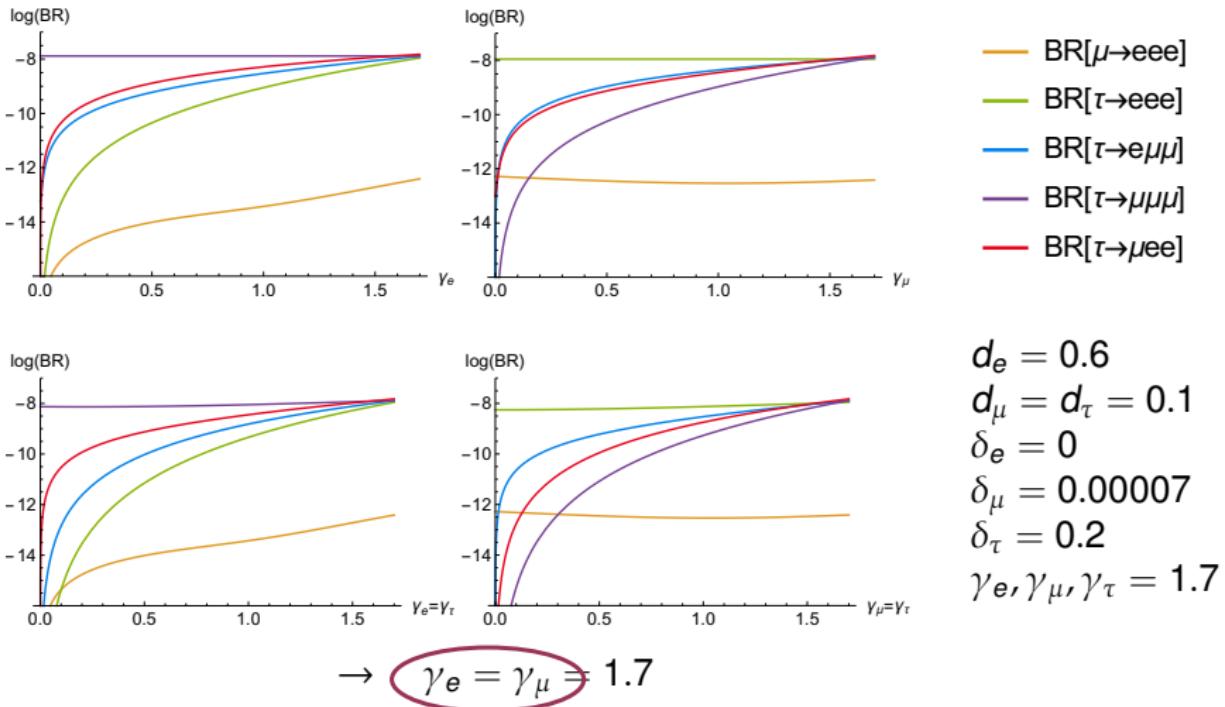
# Constrain from $4\ell$ -decays

We want  $\text{BR} [\tau^- \rightarrow \ell^- e^+ e^-] (\gamma_e, \gamma_\mu, \gamma_\tau)$  and  $\text{BR} [\tau^- \rightarrow \ell^- \mu^+ \mu^-] (\gamma_e, \gamma_\mu, \gamma_\tau)$  close to their experimental upper bounds ( $< 2.7 \times 10^{-8}$ ):



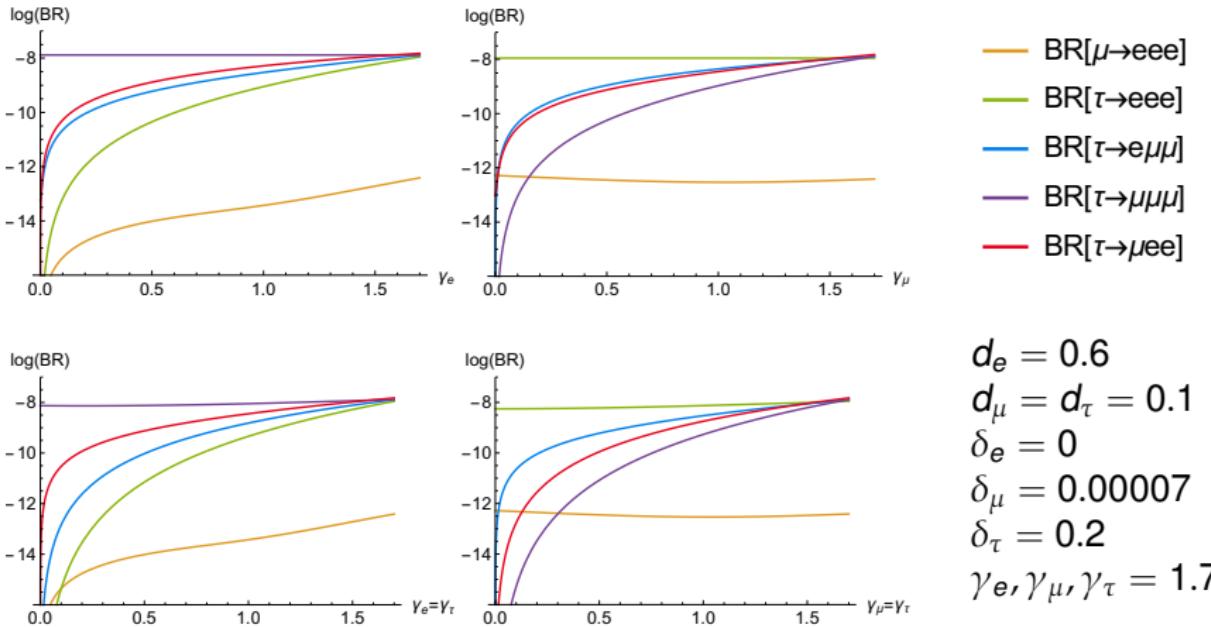
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$$\begin{aligned} d_e &= 0.6 \\ d_\mu &= d_\tau = 0.1 \\ \delta_e &= 0 \\ \delta_\mu &= 0.00007 \\ \delta_\tau &= 0.2 \\ \gamma_e, \gamma_\mu, \gamma_\tau &= 1.7 \end{aligned}$$

$$\rightarrow \gamma_e = \gamma_\mu = 1.7 \quad \checkmark \text{MDM } (\gamma_\mu \geq 1.7, \gamma_e < 2.76 \text{ TeV})$$

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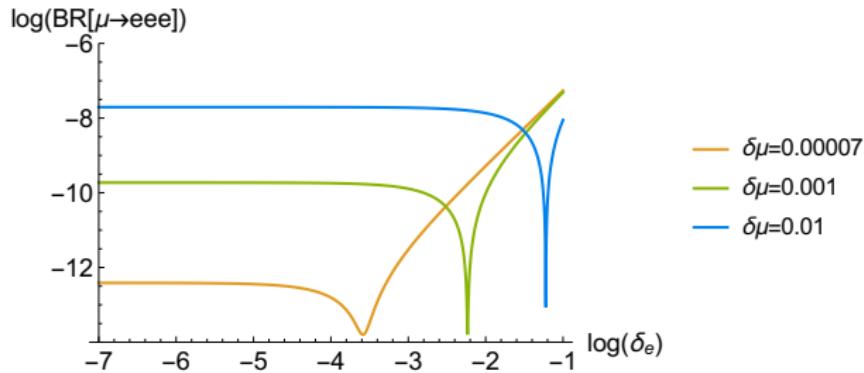
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$$\text{BR}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}$$



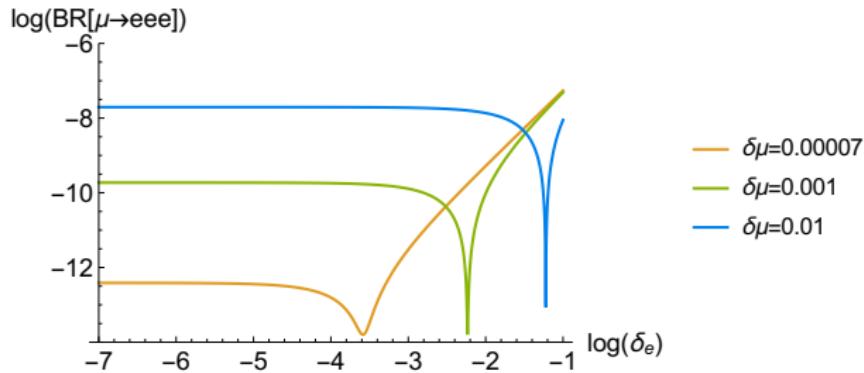
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no finetuning with:

$$\delta_e = 0$$

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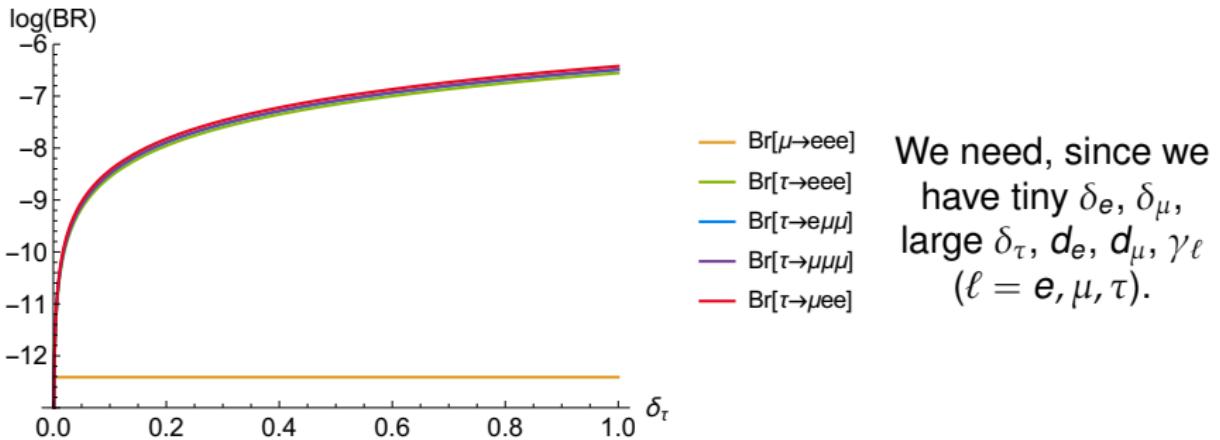
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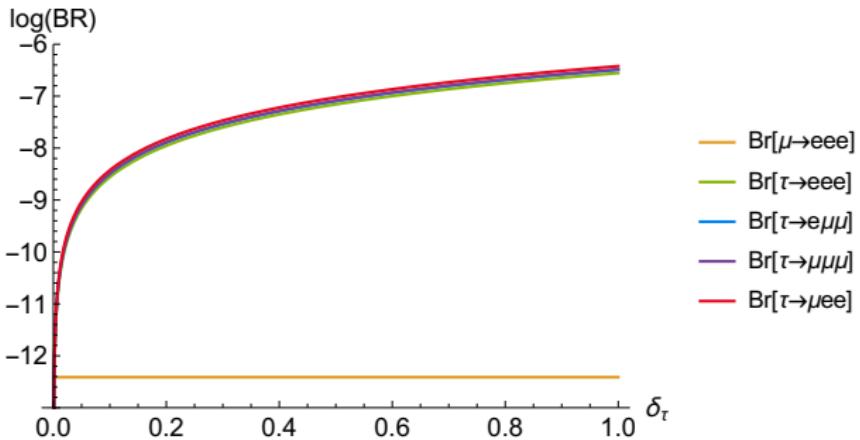
We need, since we have tiny  $\delta_e, \delta_\mu$ , large  $\delta_\tau, d_e, d_\mu, \gamma_\ell$  ( $\ell = e, \mu, \tau$ ).

e.g.

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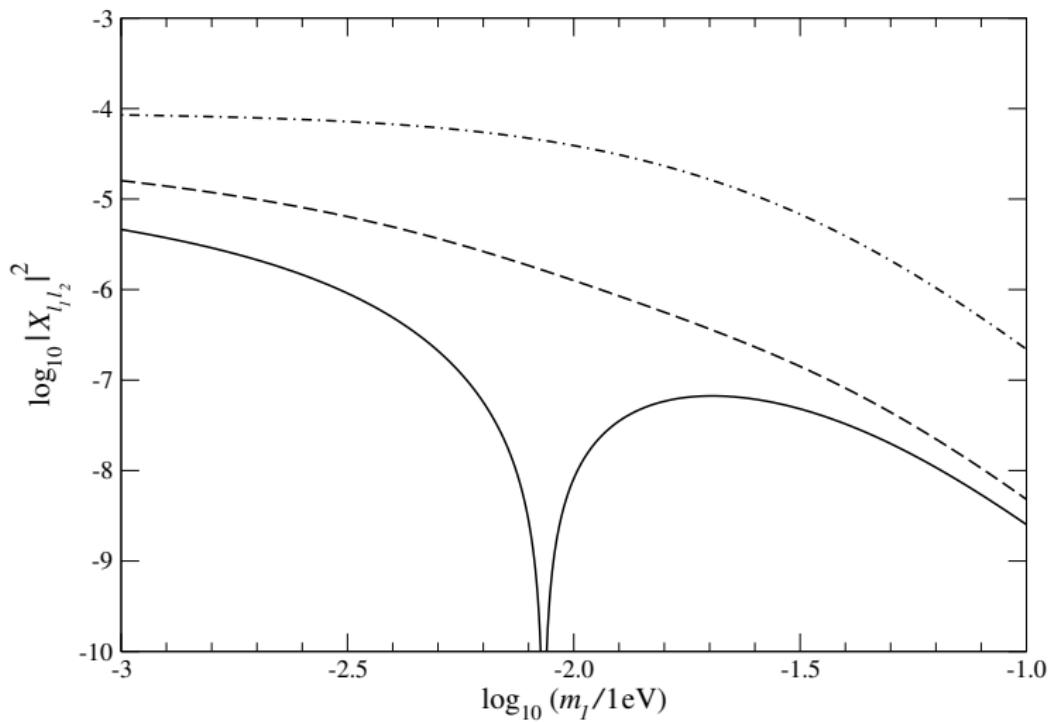
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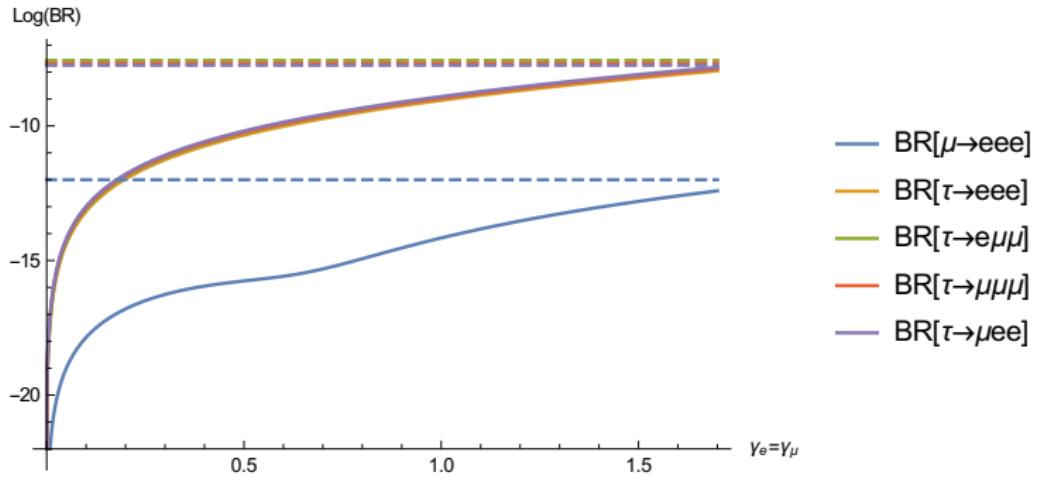
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$$\begin{aligned} \delta_\tau &= 0.2 \\ d_e &= 0.6, d_\mu = 0.1 \\ \gamma_\ell &= 1.7 \end{aligned}$$





# Advantage of right handed neutrinos

- **Explain mass hierarchy** in right handed neutrino mass models via the seesaw mechanism. [ $m_{\nu_R} \gtrsim \text{TeV}$ ] (with additional Higgs doublets...)
- **Dark matter candidates** [ $\text{keV} \lesssim m_{\nu_R} \lesssim \text{TeV}$ ]
- **Baryon asymmetry** via Leptogenesis in  $\nu$ MSM models [ $\text{keV} \lesssim m_{\nu_R} \lesssim \text{GeV}$ ]
- **Detected anomalies** at: LSND, MiniBooNE, gallium detectors: GALLEX, SAGE, reactor experiments... [ $m_{\nu_R} \sim \text{eV}$ ] (a.o. also IceCube)

tightest constraints from cosmology:

- **Boundaries from BBN**
- **CMB measurement** from PLANCK sets limits on  $N_\nu$  and also the **Large Scale Structure**.

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